Future Value of an Annuity

An annuity is a series of equal deposits made at equal time intervals.
Each deposit is made at the end of each time interval.

Example # 1:

Ethan deposited $500 into a savings account at the end of each year. He began on his 13th birthday. The account pays 6%/a. compounded annually. What is the balance in the account on Ethan's 18th birthday.

On Ethan's 18th birthday:
The $500 deposited at age 13 has earned interest for 5 years.
The amount is $500(1.06)^5$
The $500 deposited at age 14 has earned interest for 4 years.
The amount is $500(1.06)^4$
etc.....

The amount of Ethan's annuity is the sum of the amounts of the deposits.
$= 500(1.06)^5 + 500(1.06)^4 + 500(1.06)^3 + 500(1.06)^2 + 500(1.06)^1 + 500$
= $3487.66

The formula for calculating the future value of an annuity is:

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

FV is the future value of the annuity after some time period
R is the **regular** deposit into the investment
i is the adjusted interest rate per compounding period
n is the adjusted number of compounding periods

Example # 2:

Paul deposits $1000 at the end of each year in an account that pays 5%/a. compounded annually. What is the amount in the account at the end of 10 years?

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$FV = \frac{1000\left[(1+0.05)^{10} - 1\right]}{0.05}$$

$$FV = 12577.89$$

Example # 3:

Annika deposited $800 in an account at the end of every 6 months for 7 years. The account paid 9%/a. compounded semi-annually. Determine the amount in the account on the date of the last deposit.

$FV = ?$

$R = \$800$

$i = \frac{0.09}{2} = 0.045$

$n = 7 \times 2 = 14$

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$FV = \frac{800\left[(1.045)^{14} - 1\right]}{0.045}$$

$$FV = 15145.69$$

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